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Hawking Radiation of Charged Particles from a Rotating Black String

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Abstract Using Damour-Ruffini method, Hawking radiation of rotating black strings is studied. Under the condition that the total energy, total angular momentum and total charge are conservative, the transition probability from initial state (energy $M+\omega$, charge $Q+e$ and angular momentum $J+m$) to final state (energy M, charge Q and angular momentum J) for black strings is derived considering the reaction of radiation particles to spacetime. That is, the probability that black strings radiate particles with energy ω, charge e and angular momentum m is obtained. The real spectrum is not a strictly pure thermal spectrum. Our result is consistent with Parikh and Wilczek's result. It satisfies the unitary principle of quantum mechanics. However, in our result there are not only the term that denotes effect of energy and charge of radiation particles but also the term that denotes effect of radiation particles angular momentum on rotating black strings angular momentum. We provide a new way for investigating radiation of black strings.

Keywords Damour-Ruffini method · Hawking radiation · Energy conversation · Angular momentum conversation · Charge conversation

1 Introduction

From the classical point of view, any objects that come into the black hole can not escape. With the substances that come into the black hole gradually increase, the black hole will become more and more massive. In 1970s, Hawking pointed out that the black hole can radiate particles [1], which set a milestone in black hole physics. The discovery of this effect not only solved the problem in black hole thermodynamics but also announced the relation among quantum mechanics, thermodynamics and gravitation. Discovering the thermodynamic properties of various black holes is an important subject of black hole physics. Vacuum fluctuation near the surface of the black hole would produce virtual particle pair. When

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the virtual particles with negative energy come into black hole via tunnel effect, the energy of the black hole will decrease. At the same time, the particle with positive energy may thread out the gravitation region outside the black hole. Equivalently, the black hole radiates a particle. Gibbons and Hawking also demonstrated that the energy spectrum of radiation is exactly thermal [2]. Hawking radiation demonstrates that the black hole is no longer an ultimate state of a star. The black hole will evolve and finally disappear. Since Hawking did not consider the reaction of radiation to spacetime, Hawking radiation spectrum is a exact black body spectrum. However, from the black body spectrum, we can obtain one parameter-temperature. Thus the black hole radiation will not take any information about matter in the black hole. It means that if the black hole completely evaporates, all black hole information including the unitary property will disappear. The information loss of the black hole means the pure quantum state will decay to mixed state. This violates the unitary principle in quantum mechanics. This is a serious challenge to the theoretical basic of quantum mechanics.

Before 2004 Hawking had believed that during the evolution process the information is not conservative and the evolution of black hole does not satisfy the unitary principle of quantum mechanics [3]. Some physicists advocate that the information should be conservative during the evolution process and the evolution of black hole satisfies the unitary principle of quantum mechanics [4]. However, the aforesaid two view points were not proved for 30 years. Until 2004, in the 17th International Conference on General Relativity and Gravition, Hawking brought a tremendous convulsion. He proposed that the information should be conservative during the black hole formation and evaporation process [5]. But Hawking did not proved strictly the fact.

In 2000, Parikh and Wilczek proposed a semiclassical method for calculating the correction spectrum of the black hole Hawking radiation [6]. In this method, the black hole Hawking radiation is understood as a sort of quantum tunneling. Potential barrier is determined by the energy of emitted particles. The key of this method is emphasizing energy conservation during the particle emission process and establishing a good coordinate system at horizon. Using this method Parikh and Wilczek have calculated the emission correction spectrum of particles through Schwarzschild black hole and Reissner-Nordstrom black hole. The result departs from the purely thermal spectrum. It satisfies unitary principle and information conservation. Subsequently, the Hawking radiation correction spectrum of axisymmetric black holes have been calculated [7–22]. And the results satisfy unitary principle and information conservation.

Parikh and Wilczek thought that the total energy of the spacetime was conserved during the black hole radiation, and the energy of the black hole can fluctuate. When [7–22] calculated the radiation spectrum of axisymmetric black holes using the tunneling method proposed by Parikh and Wilczek, they only considered that the energy and charge fluctuated during the radiation process, the change of the black hole angular momentum was determined by the change of the black hole energy. They did not consider the effect of the rotation of the black hole radiated particles on the black hole angular momentum. References [23] started from the classical Damour-Ruffini method [24], and took the radiation process of energy as a integral process. Summing the energy of radiation particles, they derived that the radiation spectrum of the black hole departs from the black body spectrum. In this paper, we also start from the classical Damour-Ruffini method and investigate Hawking radiation of rotating black strings. Under the condition that the total energy, total angular momentum and total charge are conservative, the probability that black strings jump from initial state (energy $M+\omega$, charge $Q+e$ and angular momentum $J+m$) to final state (energy M, charge Q and angular momentum J) is derived. That is, the probability that black strings radiate particles with energy ω, charge e and angular momentum m is obtained. The result is not a strictly pure thermal spectrum. Though our result is consistent with Parikh and Wilczek's result, the physics meaning that it includes differ from Parikh and Wilczek's result. Our work is a meaningful work to help people realize black hole or black strings radiation.

2 Klein-Gordon Equations

The solution of the Einstein-Maxwell equations with a positive cosmological constant which has cylindrical symmetry can be written as [25, 26]

$$ds^2 = -\Xi^2\left(f(r) - \frac{a^2r^2}{\Xi^2 l^4}\right)dt^2 + \frac{1}{f(r)}dr^2 + \frac{r^2}{l^2}dz^2 - 2\frac{a\,\Xi l}{r}\left(b - \frac{l}{r}\gamma^2\right)dt\,d\varphi + \left[\Xi^2 r^2 - a^2 f(r)\right]d\varphi^2, \tag{1}$$

$$A_\mu = -i\,\Xi\frac{l\gamma}{r}\left(\delta_\mu^0 + \frac{a}{\Xi}\delta_\mu^2\right), \tag{2}$$

where

$$f(r) = -\left(\frac{r^2}{l^2} - \frac{bl}{r} + \frac{\gamma^2 l^2}{r^2}\right), \qquad \Xi^2 = 1 + \frac{a^2}{l^2}, \tag{3}$$

a, b, and γ are the constant parameters of the metric. It is worthwhile to mention that for the case of $-\infty < z < \infty$, (1)–(3) describe a stationary black string with cylindrical horizon.

On z axis, the mass, charge and angular momentum of per unit height are respectively [26]

$$M = \frac{1}{8}\left(3\Xi^2 - 1\right)b, \qquad Q = \frac{\Xi\gamma}{2}, \qquad J = \frac{3}{8}\Xi ab. \tag{4}$$

Again the Hawking temperature and the angular velocity of the event horizon can be calculated as

$$T_+ = \frac{f'(r_+)}{4\pi\,\Xi} = -\frac{3r_+^4 - \gamma^2 l^4}{4\pi\,\Xi l^2 r_+^3}, \qquad \Omega = \frac{a}{\Xi l^2}, \tag{5}$$

where r_+ is the position of the black string event horizon and satisfies $f(r_+) = 0$. The black string horizon area per unit length of the string for the case of a cylindrical horizon is $2\pi\,\Xi r_+^2/l$. The entropy per unit length is $\pi\,\Xi r_+^2/2l$.

In curved spacetime, Klein-Gordon equation of charged particle is

$$\frac{1}{\sqrt{-g}}\left[\left(\frac{\partial}{\partial x^\mu} - ieA_\mu\right)\sqrt{-g}g^{\mu\nu}\left(\frac{\partial}{\partial x^\nu} - ieA_\nu\right)\Phi\right] - \mu_0^2\Phi = 0, \tag{6}$$

where μ_0 is static mass of scalar particle, e is the charge of radiation particles. According to (1), the metric determinant is

$$g = -\frac{r^4}{l^2}, \tag{7}$$

and the contravariant components of the metric tensor are

$$
\begin{aligned}
&g^{00} = -\left[\frac{\Xi^2 r^2 - a^2 f(r)}{r^2 f(r)}\right], \qquad g^{11} = f(r), \qquad g^{22} = \frac{\Xi^2}{r^2 f(r)}\left[f(r) - \frac{a^2 r^2}{\Xi^2 l^4}\right], \\
&g^{33} = \frac{l^2}{r^2}, \qquad g^{02} = -\frac{a\Xi l}{r^3 f(r)}\left[b - \frac{l}{r}\gamma^2\right].
\end{aligned} \tag{8}
$$

Substituting metric (7) and (8) into (6), we have

$$
\begin{aligned}
&\left[g^{00}\frac{\partial^2}{\partial t^2} - 2ieA_0 g^{00}\frac{\partial}{\partial t} + g^{11}\frac{\partial^2}{\partial r^2} + \frac{1}{\sqrt{-g}}\left(\sqrt{-g}g^{11}\right)'\frac{\partial}{\partial r} + g^{22}\frac{\partial^2}{\partial \varphi^2} - 2ieA_0 g^{22}\frac{\partial}{\partial \varphi}\right]\Phi \\
&\quad = \left[g^{00}e^2A_0^2 + g^{22}e^2A_2^2 + \mu_0^2\right]\Phi.
\end{aligned} \tag{9}
$$

In (9) we have used the symmetry of the spacetime. There is no relation between the wave function and z. Separate variable and let $\Phi = e^{-i(\omega t - m\varphi)}R(r)$, we have

$$
\begin{aligned}
&\left[g^{11}\frac{\partial^2}{\partial r^2} + \frac{1}{\sqrt{-g}}\left(\sqrt{-g}g^{11}\right)'\frac{\partial}{\partial r}\right]R(r) \\
&\quad = \left[g^{00}\omega^2 + g^{00}2eA_0\omega + g^{22}m^2 - g^{22}2emA_2 + g^{00}e^2A_0^2 + g^{22}e^2A_2^2 + \mu_0^2\right]R(r) \\
&\qquad - 2g^{02}\left[\omega m - e\omega A_2 + emA_0 - e^2A_0A_2\right]R(r),
\end{aligned} \tag{10}
$$

$$
dr_* = \frac{1}{f(r)}dr, \qquad \frac{d}{dr} = \frac{1}{f(r)}\frac{d}{dr_*}, \tag{11}
$$

$$
\frac{d^2}{dr^2} = \frac{1}{f^2(r)}\frac{d^2}{dr_*^2} - \frac{f'(r)}{f^2(r)}\frac{d}{dr_*}. \tag{12}
$$

Then near the horizon, (10) can be reduced to

$$
\begin{aligned}
\frac{d^2R(r)}{dr_*^2} &= \left[-\omega^2\Xi^2 + 2e\omega\Xi^3\frac{l\gamma}{r_+} - m^2\frac{a^2}{l^4} + 2em\frac{a^3\gamma}{r_+} - e^2\Xi^4\frac{l^2\gamma^2}{r_+^2}\right]R(r) \\
&\quad + \left[-e^2\frac{a^4\gamma^2}{l^2r_+^2} + 2\omega m\Xi\frac{a}{l^2} - 2em\Xi\frac{a^2\gamma}{lr_+} - 2em\Xi^2\frac{a\gamma}{lr_+} + 2e^2\Xi^2\frac{a^2\gamma^2}{r_+^2}\right]R(r) \\
&= -\Xi^2\left[\omega^2 + m^2\frac{a^2}{\Xi^2l^4} + e^2\frac{l^2\gamma^2}{\Xi^2r_+^2} - 2\omega m\frac{a}{\Xi l^2} - 2e\omega\frac{l\gamma}{\Xi r_+} + 2em\frac{a\gamma}{\Xi^2 lr_+}\right]R(r) \\
&= -\Xi^2\left[\omega - m\frac{a}{\Xi l} - e\frac{l\gamma}{\Xi r_+}\right]^2 R(r).
\end{aligned} \tag{13}
$$

The solution of (13) is

$$
R = e^{\pm i\Xi(\omega-\omega_0)r_*}, \tag{14}
$$

where $\omega_0 = m\Omega + e\phi$, $\phi = \frac{l\gamma}{\Xi r_+}$ is electric potential [26].

Thus radial wave is

$$
\Psi = e^{-i\omega t \pm i\Xi(\omega-\omega_0)r_*}. \tag{15}
$$

Letting $\hat{r} = \frac{\omega-\omega_0}{\omega} r_*$, we obtain the ingoing wave at surface of the horizon

$$\Psi_{in} = e^{-i\omega(t+\Xi\hat{r})} = e^{-i\omega v}, \tag{16}$$

and outgoing wave is

$$\Psi_{out}(r > r_+) = e^{-i\omega(t-\Xi\hat{r})} = e^{-i\omega v} e^{2i\omega\Xi\hat{r}} = e^{-i\omega v} e^{2i\Xi(\omega-\omega_0)r_*}. \tag{17}$$

Where $v = t + \Xi\hat{r}$ is Eddington-Finkelstein coordinate. Because $\frac{dr}{f(r)} = dr_*$, near the horizon surface r_+ we have

$$\ln(r - r_+) = f'(r_+) r_* = 2\Xi\kappa_h r_*, \tag{18}$$

where

$$\kappa_h = \frac{f'(r_+)}{2\Xi}, \tag{19}$$

is gravity acceleration on the horizon surface r_+, $f'(r_+) = \frac{df(r)}{dr}|_{r=r_+}$. From(18), we have

$$(r - r_+) = \exp(2\kappa_h \Xi r_*), \tag{20}$$

then outgoing wave can be rewritten as

$$\Psi_{out}(r > r_+) = e^{-i\omega v}(r - r_+)^{\frac{i}{\kappa_h}(\omega-\omega_0)}. \tag{21}$$

Because the outgoing wave is singular at the horizon surface r_+, (21) only can describe the outgoing particles out of the horizon r_+ and can not describe the outgoing particles in horizon.

3 Analytic Extension

When investigating black hole string radiation, we are interested in the outgoing wave. However, from (21), the outgoing wave is singular at $r = r_+$. So we analytically extend Ψ_{out} to the inner of horizon. We take the singularity $r = r_+$ as the center of a circle, and take $|r - r_+|$ as radius. By analytical continuation rotating $-\pi$ through the lower-half complex r plane [24]

$$r - r_+ \to |r - r_+| e^{-i\pi} = (r_+ - r)e^{-i\pi}, \tag{22}$$

and derive the outgoing wave in horizon surface r_+

$$\begin{aligned} \Psi_{out}(r < r_+) &= e^{-i\omega v}\,(r_+ - r)^{i(\omega-\omega_0)/\kappa_h}\, e^{\pi(\omega-\omega_0)/\kappa_h} \\ &= e^{\pi(\omega-\omega_0)/\kappa_h} e^{-i\omega v} e^{2i\Xi(\omega-\omega_0)r_*}. \end{aligned} \tag{23}$$

Equations (21) and (23) describe respectively outgoing wave of inner and outer of black string. On black string horizon surface, the outgoing rate of outgoing wave of particles with energy ω, charge e and angular momentum m is [27]

$$\Gamma = \left|\frac{\Psi_{out}(r > r_+)}{\Psi_{out}(r < r_+)}\right|^2 = e^{-2\pi(\omega-\omega_0)/\kappa_h}. \tag{24}$$

4 Radiation Spectrum of Black String

Based on the above analysis, we know that on z axis per unit height black string with energy M, charge Q and angular momentum J can radiate particles with energy ω, charge e and angular momentum m. So the total energy, total charge and total angular momentum of per unit height spacetime on z axis are respectively $M+\omega$, $Q+e$ and $J+m$. However, the energy, charge and angular momentum of particles are taken from black string during black string radiation process. So before black string radiation, on z axis per unit height black string has energy $M+\omega$, charge $Q+e$ and angular momentum $J+m$. Therefore (24) can be taken as the probability that black strings jump from initial state (energy $M+\omega$, charge $Q+e$ and angular momentum $J+m$) to final state (energy M, charge Q and angular momentum J). If in (24) the energy ω, charge e and angular momentum m of radiation particles are described employing the parameters of black string, the result will embody the reaction of radiation to spacetime. When we describe using the parameters of black string, we should make in the z axis total energy, total charge and total angular momentum of per unit height to be conserved. That is

$$\begin{aligned} -\omega &= M-(M+\omega)=\Delta M, \\ -e &= Q-(Q+e)=\Delta Q, \\ -m &= J-(J+m)=\Delta J, \end{aligned} \tag{25}$$

where ΔM, ΔQ and ΔJ are respectively the change of energy, charge and angular momentum before and after of black string radiation respectively.

Substituting (26) into (24), we derive the outgoing rate of radiation particles described employing the parameters of the black string

$$\Gamma = \exp\left[(\Delta M - \phi\Delta Q - \Omega_+\Delta J)/T_+\right], \tag{26}$$

where $T_+ = \frac{\kappa_h}{2\pi}$ is Hawking radiation temperature of black string. According to the first law of thermodynamics of black string [26, 28, 29]

$$\Delta M = T_+\Delta S + \phi\Delta Q + \Omega_+\Delta J, \tag{27}$$

and (26), we obtain

$$\Gamma = e^{\Delta S}, \tag{28}$$

where

$$\begin{aligned} \Delta S = {} & \frac{\pi}{2l}\Xi(M,Q,J)r_+^2(M,Q,J) \\ & - \frac{\pi}{2l}\Xi(M+\omega,Q+e,J+m)r_+^2(M+\omega,Q+e,J+m) \end{aligned} \tag{29}$$

is the change in the Bekenstein-Hawking entropy before and after the radiation. Our result is consistent with the result of Parikh and Wilczek. It satisfies the unitary principle of quantum mechanics. However, our method is simple and straight.

5 Conclusion and Discussion

Based on the above calculation, our result (28) has the same form with the results in literatures [7–22]. However, in Bekenstein-Hawking entropy difference expression (28) there is the term that denotes effect of angular momentum of radiation particles on black strings angular momentum. That is the angular momentum of black string become J from $(J+m)$. References [7–22] only considered the effect of energy charge on the black hole angular momentum. They did not consider the effect of radiation particle rotation on black hole angular momentum. So our result has general meaning. Though our result is consistent with the results of Parikh and Wilczek and [30–38] and also satisfies the unitary principle of quantum mechanics, we start from the classical Damour-Ruffini method and investigate Hawking radiation of black strings considering the reaction of radiation to spacetime. Under the condition that the total energy, total angular momentum and total charge are conservative, the result that the radiation spectrum departs from the spectrum of black body is obtained. Our method is different from the one of [6, 23]. In our calculation, we need not consider whether radiation particles have static mass. So our method is simple.

In this paper, we adapt the same method as the one of [23] and also start from the classical Damour-Ruffini method. On black string horizon surface, the outgoing rate of outgoing wave of particles with energy ω, charge e and angular momentum m is given by (24) ((19) in [23]). Reference [23] took the radiation process of energy as a integral process. Summing the energy of radiation particles, they derived that the radiation spectrum of the black hole departs from the black body spectrum. In this paper, under the condition that the total energy, total angular momentum and total charge are conservative, the probability that black Strings jump from initial state (energy $M+\omega$, charge $Q+e$ and angular momentum $J+m$) to final state (energy M, charge Q and angular momentum J) is derived. That is, the probability that black strings radiate particles with energy ω, charge e and angular momentum m is obtained.

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